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Quantum dissipation is studied within two model oscillators, the Caldirola-Kanai (CK) oscillator as an open system with one degree of freedom and the Bateman-Feshbach-Tikochinsky (BFT) oscillator as a closed system with two degrees of freedom. Though these oscillators describe the same classical damped motion, the CK oscillator retains the quantum coherence, whereas the damped subsystem of the BFT oscillator exhibits both quantum decoherence and classical correlation. Furthermore the amplified subsystem of the BFT oscillator shows the same degree of quantum decoherence and classical correlation.

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I. INTRODUCTION

Physical systems are believed without doubt to obey quantum theory that has been tested in various areas. Microscopic systems are described most accurately by the quantum theory, whereas most of the macroscopic systems are described also by classical theory to any desired precision. The most prominent feature of quantum theory is quantum coherence, the interference among superposed states. Then the question may be raised: how quantum states of a system can have the characteristic features of classical theory such as quantum decoherence (loss of quantum coherence) and classical correlation along classical trajectory? This quantum-to-classical transition or classicality of quantum system has been an important problem since the advent of quantum theory. Though not completely settled, it is well known that the interaction of a quantum system with an environment can lead to classicality of quantum decoherence and classical correlation [1–8]. In particular, dissipation is known to result in quantum decoherence [5–7]. In most of literature, the quantum decoherence has been studied for a system coupled to an environment or thermal bath with many degrees of freedom.

In this paper we study quantum damped oscillators as a simple model for the dissipative system. There are different Hamiltonian representations for this damped oscillator. One representation is the Caldirola-Kanai (CK) oscillator, which is a one-dimensional system with an exponentially increasing mass [9,10]. This oscillator is an open system because its parameters such as mass or frequency depend explicitly on time. The other representation is the Bateman or Feshbach-Tikochinsky (BFT) oscillator, which consists of a damped oscillator and an amplified oscillator [11–13]. The second oscillator is a closed system as the total energy is conserved and the energy dissipated from the damped oscillator is transferred to the amplified one. These quantum damped oscillators have been studied intensively as a model to understand dissipation in quantum theory and the connection between them has been found [14]. The CK oscillator has also been investigated to find the characteristic features of quantum states [15–18] (for review and references, see Ref. [19]). The BFT oscillator has also been used to study quantum dissipation [13,14,20–23].

The main purpose of this paper is to study the classicality of the damped CK and BFT oscillators as a quantum mechanical system with dissipation and few degrees of freedom. More concretely we investigate quantum decoherence, the necessary condition, and classical correlation, the sufficient condition for classicality. For that purpose we first find the density matrix of the Gaussian state for the CK oscillator and the reduced density matrix for the damped part of the BFT oscillator and then apply the criterion on measuring quantitatively quantum decoherence and classical

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correlation. The measure of quantum decoherence is defined as the ratio of the diagonal element to the off-diagonal element of the (reduced) density matrix [24]. It is found that the density matrix for the CK oscillator shows no quantum decoherence, whereas the reduced density matrix for the BFT oscillator shows both quantum decoherence and classical correlation for the damped and amplified parts.

The organization of this paper is as follows. In Sec. II we find the quantum state and density matrix of CK oscillator and calculate the measure of quantum decoherence and classical correlation. In Sec. III we find the density matrix and thereby the reduced density matrix of the BFT oscillator. The measure of decoherence is evaluated. In Sec. IV we study the amplified oscillator, the opposite case of damped oscillator and investigate the condition for classicality.

II. CK OSCILLATOR

The CK oscillator is an an open system with the variable mass $m(t) = me^{\gamma t/m}$ [9,10]

$$H_{\text{CK}} = \frac{1}{2me^{\gamma t/m}} p_x^2 + \frac{m\omega^2 e^{\gamma t/m}}{2} x^2, \quad (1)$$

and has the classical equation of motion for dissipative system

$$\ddot{x} + \frac{\gamma}{m} \dot{x} + \omega^2 x = 0. \quad (2)$$

It is assumed that the quantum theory of the damped oscillator is prescribed by the time-dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = \hat{H}_{\text{CK}}(t) \Psi(x, t). \quad (3)$$

The wave functions are found in various methods [15–18] (for review and references, see Ref. [19]). Lewis and Riesenfeld introduced an invariant operator for the general time-dependent oscillator, whose eigenstate is an exact quantum state up to a time-dependent phase factor [25].

Following Refs. [26–28], we introduce a pair of first order operators in position and momentum

$$\begin{aligned} \hat{a}(t) &= i[u^*(t)\hat{p}_x - \dot{u}^*(t)\hat{x}], \\ \hat{a}^\dagger(t) &= -i[u(t)\hat{p}_x - \dot{u}(t)\hat{x}], \end{aligned} \quad (4)$$

and require them to satisfy the quantum Liouville-von Neumann equation

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \hat{a}(t) + [\hat{a}(t), \hat{H}_{\text{CK}}(t)] &= 0, \\ i\hbar \frac{\partial}{\partial t} \hat{a}^\dagger(t) + [\hat{a}^\dagger(t), \hat{H}_{\text{CK}}(t)] &= 0. \end{aligned} \quad (5)$$

Then u satisfies the classical equation of motion (2). The Wronskian condition

$$\hbar m e^{\gamma t/m} [\dot{u}^*(t)u(t) - \dot{u}(t)u^*(t)] = i, \quad (6)$$

guarantees the standard commutation relation for all times

$$[\hat{a}(t), \hat{a}^\dagger(t)] = 1. \quad (7)$$

We are interested in the underdamped motion given by

$$u(t) = \frac{1}{\sqrt{2\hbar m \Omega}} e^{-\gamma t/(2m)} e^{-i\Omega t}, \quad (8)$$

where

$$\Omega = \sqrt{\omega^2 - \left(\frac{\gamma}{2m}\right)^2}, \quad (\gamma \leq 2m\omega). \quad (9)$$

The number operator defined by

$$\hat{N}(t) = \hat{a}^\dagger(t)\hat{a}(t) \quad (10)$$

also satisfies Eq. (5) and yields the number state as an exact quantum state

$$\hat{N}(t)|n, t\rangle = n|n, t\rangle. \quad (11)$$

The wave function for the number state that satisfies Eq. (3) is given by [28]

$$\Psi_n(x, t) = \left(\frac{m\Omega e^{\gamma t/m}}{\pi\hbar}\right)^{1/4} \frac{e^{-i(n+1/2)\Omega t}}{\sqrt{2^n n!}} H_n\left(\sqrt{\frac{m\Omega e^{\gamma t/m}}{\hbar}} x\right) \exp\left[-e^{\gamma t/m} \left(\frac{m\Omega}{2\hbar} + i\frac{\gamma}{4\hbar}\right) x^2\right], \quad (12)$$

where H_n is the Hermite polynomial. The wave function has the dispersion relations

$$\begin{aligned} \langle \hat{x}^2 \rangle &= \hbar^2 u^*(t)u(t) = \frac{\hbar}{2m\Omega} e^{-\gamma t/m}, \\ \langle \hat{p}_x^2 \rangle &= \hbar^2 m^2(t) \dot{u}^* \dot{u} = \frac{\hbar m \omega^2}{2\Omega} e^{\gamma t/m}. \end{aligned} \quad (13)$$

The more the system dissipates in time, the more the wave function becomes sharply peaked around $x = 0$, whereas the wave function is more dispersed in momentum space. The uncertainty is a constant

$$(\Delta x)(\Delta P_x) = \frac{\hbar\omega}{2\Omega} \geq \frac{\hbar}{2}, \quad (14)$$

and the Hamiltonian expectation value is also a constant.

$$\langle 0, t | \hat{H}_{\text{CK}}(t) | 0, t \rangle = \frac{\hbar\omega^2}{\Omega} \geq \frac{\hbar\omega}{2}. \quad (15)$$

The density matrix of the Gaussian wave function (12), the ground state with $n = 0$, has the form

$$\begin{aligned} \rho_{\text{CK}}(x', x, t) &= \Psi_0(x')\Psi_0^*(x) \\ &= \left(\frac{m\Omega e^{\gamma t/m}}{\pi\hbar}\right)^{1/2} \exp\left[-\Gamma_c x_c^2 - \Gamma_\delta x_\delta^2 - \Gamma_\mu x_c x_\delta\right], \end{aligned} \quad (16)$$

where

$$x_c = \frac{1}{2}(x' + x), \quad x_\delta = \frac{1}{2}(x' - x), \quad (17)$$

and

$$\Gamma_c = \Gamma_\delta = \frac{m\Omega}{\hbar} e^{\gamma t/m}, \quad \Gamma_\mu = -i\frac{\gamma}{\hbar} e^{\gamma t/m}. \quad (18)$$

The off-diagonal element, the coefficient of x_δ^2 , measures the degree of quantum coherence, *i.e.*, the interference between two different trajectories. The representation-independent measure of quantum decoherence [24] is now given by

$$\delta_{\text{QD}} = \frac{1}{2} \sqrt{\frac{\Gamma_c}{\Gamma_\delta}} = \frac{1}{2}. \quad (19)$$

The condition for quantum decoherence ($\delta_{\text{QD}} \ll 1$) shows no decoherence for the CK oscillator. Likewise, the measure of classical correlation

$$\delta_{\text{CC}} = \sqrt{\frac{\Gamma_c^2 \Gamma_\delta^2}{\Gamma_\mu^* \Gamma_\mu}} = \frac{(m\Omega)^2}{\hbar\gamma} e^{\gamma t/m}, \quad (20)$$

shows no classical correlation conditioned by $\delta_{\text{CC}} \ll 1$, except for the case of $\Omega \approx 0$ in the large-damping limit $\gamma \approx 2m\omega$. Even in this case the exponentially growing factor dominates at later times and classical correlation is lost. The zero-damping limit ($\gamma = 0$) of a pure harmonic oscillator does not lead to any classical correlation with the infinite δ_{CC} as expected. The CK oscillator does achieve neither quantum decoherence nor classical correlation.

In summary, the CK oscillator does not have the genuine properties of dissipative systems, though its equation of motion does show such a damping effect. First, the energy defined by the expectation value of the Hamiltonian operator does not have any damping factor that implies the dissipation of energy. Second, the uncertainty does not grow as the evolution proceeds. Third, there is neither quantum decoherence nor classical correlation regardless of the magnitude of damping factor.

The one-dimensional CK oscillator has a constant expectation value of the Hamiltonian. To be a genuine dissipative system, the energy of the damped subsystem of the system must be dissipated away and transferred to another subsystem. This means that the damped oscillator may be properly described by a two-dimensional system, one subsystem of which dissipates the energy and the transferred energy amplifies the other subsystem. Such a model has been suggested long ago by Bateman [11] and later by Feshbach and Tikochinsky [12,13]. The BFT oscillator is described by the Lagrangian

$$L = m\dot{x}\dot{y} + \frac{\gamma}{2}(x\dot{y} - \dot{x}y) - kxy. \quad (21)$$

The one subsystem with x variable obeys the damped equation of motion (2) where $\omega = \sqrt{k/m}$. The other subsystem obeys the equation for an amplified oscillator

$$\ddot{y} - \frac{\gamma}{m}\dot{y} + \omega^2 y = 0. \quad (22)$$

The energy of y increases as $e^{\gamma t/m}$. The Hamiltonian is given by

$$H_\gamma = \frac{1}{m}p_x p_y + \frac{\gamma}{2m}(y p_y - x p_x) + \Omega^2 x y, \quad (23)$$

where $p_x = m\dot{y}$ and $p_y = m\dot{x}$.

In the limit of zero-dissipation ($\gamma = 0$), the BFT oscillator is the sum of two decoupled oscillators with opposite signs

$$H_0 = \frac{1}{2m}p_\xi^2 + \frac{k}{2}\xi^2 - \frac{1}{2m}p_\zeta^2 - \frac{k}{2}\zeta^2, \quad (24)$$

where

$$\xi = \frac{1}{\sqrt{2}}(x + y), \quad \zeta = \frac{1}{\sqrt{2}}(-x + y). \quad (25)$$

The ground state of each oscillator leads to the zero energy and a density matrix

$$\begin{aligned} \rho(x', y', x, y) &= \left(\frac{m\omega}{\pi\hbar}\right) \exp\left[-\frac{m\omega}{2\hbar}(\xi'^2 + \zeta'^2 + \xi^2 + \zeta^2)\right] \\ &= \left(\frac{m\omega}{\pi\hbar}\right) \exp\left[-\frac{m\omega}{2\hbar}(x'^2 + y'^2 + x^2 + y^2)\right]. \end{aligned} \quad (26)$$

For the dissipation case ($\gamma \neq 0$), the density matrix satisfies the quantum Liouville-von Neumann equation, whose coordinate representation is given by

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \rho(x', y', x, y) &= \left[-\frac{\hbar^2}{m} \left(\frac{\partial^2}{\partial x' \partial y'} - \frac{\partial^2}{\partial x \partial y} \right) + i\frac{\hbar\gamma}{2m} \left(x' \frac{\partial}{\partial x'} - y' \frac{\partial}{\partial y'} + x \frac{\partial}{\partial x} - y \frac{\partial}{\partial y} \right) \right. \\ &\quad \left. + m\Omega^2(x'y' - xy) \right] \rho(x', y', x, y). \end{aligned} \quad (27)$$

The density matrix quadratic in x', y', x, y has the general form

$$\begin{aligned} \rho(x', y', x, y) &= N \exp \left[-e^{\gamma t/m} (A^* x'^2 + A_1 x' x + A x^2) - e^{-\gamma t/m} (B^* y'^2 + B_1 y' y + B y^2) \right. \\ &\quad \left. - C(x'y' + xy) - (Dx'y + D^*xy') \right], \end{aligned} \quad (28)$$

where the coefficients satisfy the set of equations

$$\begin{aligned} \dot{A} &= i\frac{\hbar}{m}(2AC - A_1 D^*), \\ \dot{B} &= i\frac{\hbar}{m}(2BC - B_1 D), \end{aligned}$$

$$\begin{aligned}
\dot{A}_1 &= i\frac{2\hbar}{m}(AD - A^*D^*), \\
\dot{B}_1 &= i\frac{2\hbar}{m}(BD^* - B^*D), \\
\dot{C} &= i\frac{\hbar}{m}(4AB + C^2 - A_1B_1 - D^*D) - i\frac{m\Omega^2}{\hbar}, \\
\dot{D} &= i\frac{2\hbar}{m}(A_1B - A^*B_1).
\end{aligned} \tag{29}$$

There is a symmetry under $x \leftrightarrow y$ and $\gamma \leftrightarrow -\gamma$, which is in fact the time-reversal symmetry of the Hamiltonian (23). Hence we can set $A = B^*$ and $A_1 = B_1^*$. Also in the zero-dissipation limit, by comparing the density matrices (26) and (28) we find that A_1, C, D approach zero. A particular solution is found to be

$$\begin{aligned}
A(\gamma) &= B^*(\gamma) = \left[\left(\frac{m\Omega}{2\hbar} \right)^2 + \frac{D^*(\gamma)D(\gamma)}{4} \right]^{1/2} e^{i(\pi-\theta)}, \\
D(\gamma) &= |D(\gamma)|e^{i\theta}, \\
A_1 &= B_1 = C = 0,
\end{aligned} \tag{30}$$

where

$$D(\gamma \rightarrow 0) \rightarrow 0. \tag{31}$$

Now the density matrix from the particular solution (30) becomes

$$\rho(x', y', x, y) = N \exp \left[-e^{\gamma t/m} (A^* x'^2 + Ax^2) - e^{-\gamma t/m} (Ay'^2 + A^* y^2) - (Dx'y + D^* xy') \right], \tag{32}$$

where

$$N = \frac{1}{\pi} \left[(A + A^*)^2 - \frac{1}{4}(D + D^*)^2 \right]^{1/2}. \tag{33}$$

Letting $y' = y$ and integrating over y , one obtains the reduced density matrix for the damped subsystem of x' and x :

$$\rho_{\text{red}}(x', x) = N_1 \exp \left[-e^{\gamma t/m} \left\{ (A^* x'^2 + Ax^2) - \frac{(Dx' + D^*x)^2}{4(A + A^*)} \right\} \right], \tag{34}$$

where

$$N_1 = N \times \sqrt{\frac{\pi e^{\gamma t/m}}{(A + A^*)}}. \tag{35}$$

Similarly, the reduced density matrix for the amplified subsystem is obtained by using the symmetry $x \rightarrow y$ and $\gamma \rightarrow -\gamma$:

$$\rho_{\text{red}}(y', y) = N_2 \exp \left[-e^{-\gamma t/m} \left\{ (Ay'^2 + A^* y^2) - \frac{(Dy + D^*y')^2}{4(A + A^*)} \right\} \right], \tag{36}$$

where

$$N_2 = N \times \sqrt{\frac{\pi e^{-\gamma t/m}}{(A + A^*)}} \tag{37}$$

Then the reduced density matrix (34) is written in the form

$$\rho_{\text{red}}(x', x) = N_1 \exp \left[-\Gamma_c x_c^2 - \Gamma_\delta x_\delta^2 - \Gamma_\mu x_c x_\delta \right], \tag{38}$$

where

$$\begin{aligned}
\Gamma_c(|D|, \theta) &= e^{\gamma t/m} \left[(A + A^*) - \frac{(D + D^*)^2}{4(A + A^*)} \right], \\
\Gamma_\delta(|D|, \theta) &= e^{\gamma t/m} \left[(A + A^*) - \frac{(D - D^*)^2}{4(A + A^*)} \right], \\
\Gamma_\mu(|D|, \theta) &= e^{\gamma t/m} (-2) \left[(A - A^*) + \frac{D^2 - D^{*2}}{4(A + A^*)} \right].
\end{aligned} \tag{39}$$

As the density matrix (32) and the reduced one (34) depend only two real parameters $|D|$ and θ , the measure of quantum decoherence is given by

$$\begin{aligned}
\delta_{\text{QD}} &= \frac{1}{2} \sqrt{\frac{\Gamma_c(|D|, \theta)}{\Gamma_\delta(|D|, \theta)}} \\
&= \frac{1}{2} \sqrt{\frac{(m\Omega/\hbar)^2}{(m\Omega/\hbar)^2 \cos^2 \theta + |D|^2}}.
\end{aligned} \tag{40}$$

and that of classical correlation by

$$\begin{aligned}
\delta_{\text{CC}} &= \sqrt{\frac{\Gamma_c^2(|D|, \theta) \Gamma_\delta^2(|D|, \theta)}{\Gamma_\mu^*(|D|, \theta) \Gamma_\mu(|D|, \theta)}} \\
&= \frac{1}{2} |\cot \theta| \left[\left(\frac{m\Omega}{\hbar} \right)^2 \cos^2 \theta + |D|^2 \right].
\end{aligned} \tag{41}$$

If the condition

$$\theta \approx \frac{\pi}{2}, \quad |D| > \frac{m\Omega}{\hbar} \tag{42}$$

is satisfied, the density matrices (32) and (34) achieve a significant degree of quantum decoherence $\delta_{\text{QD}} < 1/2$ and a sufficient degree of classical correlation $\delta_{\text{CC}} \ll 1$. This is the case of the large dissipation.

In summary, the BFT oscillator has the density matrix which achieves quantum decoherence as well as almost complete classical correlation. This means that the BFT oscillator may be the quantum analog of a classical dissipative oscillator.

IV. AMPLIFIED OSCILLATOR

We now consider the opposite case of the damped oscillator, that is, the amplified oscillator, which is physically motivated by an unstable system, for instance, the second order phase transition during the spinodal instability. Quantum decoherence has not been observed for an unstable, exponentially growing, single oscillator [29]. This oscillator has the Hamiltonian

$$H_y = \frac{1}{2m e^{-\gamma t/m}} p_y^2 + \frac{m\omega^2 e^{-\gamma t/m}}{2} y^2. \tag{43}$$

The time-dependent annihilation and creation operators are given by

$$\begin{aligned}
\hat{b}(t) &= i[v^*(t)\hat{p}_y - \dot{v}^*(t)\hat{y}], \\
\hat{b}^\dagger(t) &= -i[v(t)\hat{p}_y - \dot{v}(t)\hat{y}],
\end{aligned} \tag{44}$$

where

$$\ddot{v} - \frac{\gamma}{m} \dot{v} + \omega^2 v = 0. \tag{45}$$

Then the exponentially growing solution is given by

$$v(t) = \frac{1}{\sqrt{2\hbar m \Omega_y}} e^{\gamma t/(2m)} e^{-i\Omega t}, \tag{46}$$

where Ω is given by Eq. (9). The harmonic oscillator wave functions are obtained from Eq. (12) by replacing γ by $(-\gamma)$:

$$\Psi_n(y, t) = \left(\frac{m\Omega e^{-\gamma t/m}}{\pi\hbar} \right)^{1/4} \frac{e^{-i(n+1/2)\Omega t}}{\sqrt{2^n n!}} H_n \left(\sqrt{\frac{m\Omega e^{-\gamma t/m}}{\hbar}} y \right) \exp \left[-e^{-\gamma t/m} \left(\frac{m\Omega}{2\hbar} - i\frac{\gamma}{4\hbar} \right) y^2 \right]. \quad (47)$$

Repeating the steps in Sec. II, the dispersion relations are given by

$$\begin{aligned} \langle \hat{y}^2 \rangle &= \frac{\hbar}{2m\Omega_y} e^{\gamma t/m}, \\ \langle \hat{p}_y^2 \rangle &= \frac{\hbar m \omega^2}{2\Omega_y} e^{-\gamma t/m}. \end{aligned} \quad (48)$$

So the uncertainty relation and the Hamiltonian expectation value have the same values as Eqs. (14) and (15), respectively. As the ground state wave function ($n = 0$) in Eq. (47) is obtained by replacing γ by $(-\gamma)$ and all the steps are the same in Sec. II, we find the measures for quantum decoherence and classical correlation

$$\begin{aligned} \delta_{\text{QD}} &= \frac{1}{2}, \\ \delta_{\text{CC}} &= \frac{(m\Omega)^2}{\hbar\gamma} e^{-\gamma t/m}. \end{aligned} \quad (49)$$

The wave functions for the amplified oscillator are certainly classically correlated as shown in Ref. [29]. But this does not mean that the wave functions achieve quantum decoherence because $\delta_{\text{QD}} = 1/2$.

Now we turn to the amplified subsystem of the BFT oscillator in Sec. III. The reduced density matrix for the amplified subsystem can be written in the form

$$\rho_{\text{red}}(y', y) = N_2 \exp \left[-\Gamma_c^y y_c^2 - \Gamma_\delta^y y_\delta^2 - \Gamma_\mu^y y_c y_\delta \right], \quad (50)$$

where

$$\begin{aligned} \Gamma_c^y(|D|, \theta) &= e^{-\gamma t/m} \left[(A + A^*) - \frac{(D + D^*)^2}{4(A + A^*)} \right], \\ \Gamma_\delta^y(|D|, \theta) &= e^{-\gamma t/m} \left[(A + A^*) - \frac{(D - D^*)^2}{4(A + A^*)} \right], \\ \Gamma_\mu^y(|D|, \theta) &= e^{-\gamma t/m} (2) \left[(A - A^*) + \frac{D^2 - D^{*2}}{4(A + A^*)} \right]. \end{aligned} \quad (51)$$

Therefore, for the particular solution (30) the measures for quantum decoherence and classical correlation for the amplified subsystem are given by the same Eqs. (40) and (41) for the damped subsystem. This may be expected from the symmetry of the Hamiltonian under $x \leftrightarrow y$ and $\gamma \leftrightarrow -\gamma$. There is also the symmetry of reduced density matrices of x and y under $\gamma \leftrightarrow -\gamma$ and $D(-\gamma) = D^*(\gamma)$. When the condition (42) is satisfied, the amplified subsystem also achieves both quantum decoherence and classical correlation. Such classicality of wave functions has been observed for the system of an unstable amplified oscillator coupled to a stable oscillator in a quantum phase transition model [29]. The BFT oscillator provides an exactly solvable model for both quantum decoherence and classical correlation and may shed some light in understanding how the classicality of quantum systems can be achieved.

V. DISCUSSION

We have studied the CK and BFT oscillators as the quantum analogs of a classical dissipative oscillator that obeys the classical damped equation of motion. The CK oscillator is a one-dimensional oscillator with an exponentially increasing mass in time. The wave function of the CK oscillator is sharply peaked around the origin with the exponentially decreasing position-dispersion. However, the momentum-dispersion increases exponentially so that the uncertainty of both position and momentum is constant and has the value greater than the minimum value of Heisenberg uncertainty relation. Similarly, the Hamiltonian expectation value has a constant value greater than the

minimum value of a harmonic oscillator without the damping factor. Further, CK oscillator does not achieve both quantum decoherence and classical correlation.

On the other hand, the BFT oscillator is a two-dimensional system, one subsystem describing the damped oscillator and the other describing an amplified oscillator. In this oscillator the energy dissipated away from the damped oscillator is transferred to the other so that the total energy is conserved. We have found two parameter-dependent density matrix (32), whose reduced density matrix for the damped oscillator or the amplified oscillator shows not only quantum decoherence but also classical correlation. The quantum decoherence of the BFT oscillator may be understood as a consequence of the coupling between the damped and amplified modes.

The fact that the interaction or coupling of the system is indispensable for quantum decoherence has been observed in the two-oscillator model with the Hamiltonian [8]

$$H = \frac{1}{2m}(p_1^2 + p_2^2) + \frac{m}{2}(\omega_1^2 x_1^2 + \omega_2^2 x_2^2) + \lambda x_1 x_2. \quad (52)$$

The reduced density matrix for x_1 is given by

$$\rho_{\text{red}}(x'_1, x_1) = \left(\frac{1}{\pi D}\right)^{1/2} \exp\left[-\frac{1}{D}(x_c^2 + \Gamma_\delta x_\delta^2)\right], \quad (53)$$

where

$$\begin{aligned} D &= \cosh \eta - \sinh \eta \cos(2\vartheta), \\ \Gamma_\delta &= \cosh^2 \eta - \sinh^2 \eta \cos^2(2\vartheta), \end{aligned} \quad (54)$$

where

$$\begin{aligned} e^\eta &= \frac{m(\omega_1^2 + \omega_2^2) + \sqrt{m^2(\omega_1^2 - \omega_2^2)^2 + 4\lambda^2}}{2\sqrt{m^2\omega_1^2\omega_2^2 - \lambda^2}}, \\ \tan(2\vartheta) &= \frac{2\lambda}{m(\omega_2^2 - \omega_1^2)}. \end{aligned} \quad (55)$$

The measure of decoherence

$$\delta_{\text{QD}} = \frac{1}{2\sqrt{\cosh^2 \eta - \sinh^2 \eta \cos^2(2\vartheta)}} \quad (56)$$

has a value $\delta_{\text{QD}} \leq 1/2$, the equality corresponding to the zero mode-mixing, and has small values $\delta_{\text{QD}} \ll 1/2$ for large mixing angles ($\vartheta \approx \pi/4$ and $\eta \gg 1$). However, the measure of classical correlation is infinite, the same as the quantum state of a simple harmonic oscillator. Therefore we may conclude that a system with dissipation (damping) interacting with another system can achieve both quantum decoherence and classical correlation.

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